Key

3.11 Sum & Difference Identities

Name______________________________ **Date** ______________ **Period** __________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use a sum or difference identity to find an exact value. Show work!

1) $\sin 15°$ $= \sin(45-30) = \sin 45° \cdot \cos 30° - \sin 30° \cdot \cos 45°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

1) $\frac{\sqrt{6}-\sqrt{2}}{4}$

2) $\sin 75°$

$\sin(45°+30°) = \sin 45° \cos 30° + \sin 30° \cdot \cos 45°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

2) $\frac{\sqrt{6}+\sqrt{2}}{4}$

3) $\cos \frac{\pi}{12}$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3} \cdot \cos \pi/4 + \sin \pi/3 \cdot \sin \pi/4$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

3) $\frac{\sqrt{2}+\sqrt{6}}{4}$

4) $\tan \frac{5\pi}{12}$

$\tan\left(\pi/4 + \frac{\pi}{6}\right) = \frac{\tan \pi/4 + \tan \pi/6}{1 - \tan \pi/4 \cdot \tan \pi/6} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}} \cdot \left(\frac{3}{3}\right) = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

4)

5) $\cos \frac{7\pi}{12}$

$= \cos(\pi/3 + \pi/4) = \cos\frac{\pi}{3} \cos \pi/4 - \sin \pi/3 \cdot \sin \pi/4$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

5) $\frac{\sqrt{2}-\sqrt{6}}{4}$

Write the expression as the sine, cosine, or tangent of an angle. Show work!

6) $\sin 42° \cos 17° - \cos 42° \sin 17°$

$\sin(42°-17°) = \sin 25°$

6) $\sin 25°$

7) $\sin \frac{\pi}{5} \cos \frac{\pi}{2} + \cos \frac{\pi}{5} \sin \frac{\pi}{2}$

$\sin\left(\frac{\pi}{5} + \frac{\pi}{2}\right) = \sin \frac{7\pi}{10}$

7) $\sin \frac{7\pi}{10}$

8) $\frac{\tan 19° + \tan 47°}{1 - \tan 19° \tan 47°}$ $= \tan(19° + 47°) = \tan 66°$

8) $\tan 66°$

9) $\cos \frac{\pi}{7} \cos x - \sin \frac{\pi}{7} \sin x$ $= \cos(\pi/7 + x)$

9) $\cos(\pi/7 + x)$

10) $\sin 3x \cos x - \cos 3x \sin x$ $= \sin(3x - x) = \sin 2x$

10) $\sin 2x$

11) $\frac{\tan 2y + \tan 3x}{1 - \tan 2y \tan 3x}$ $= \tan(2y + 3x)$

11) $\tan(2y + 3x)$

Use sum or difference identities (not calculator) to solve the equation exactly. Show work!

12) $\sin 2x \cos x = \cos 2x \sin x$

12) ____________

$\sin 2x \cos x - \cos 2x \sin x = 0$

$\sin(2x - x) = 0$

$\sin x = 0$

$x = 0, \pi$

Prove the identity. Show work!

13) $\sin\left(\frac{\pi}{2} - u\right) = \cos u$

13) ____________

$\sin\frac{\pi}{2} \cdot \cos u - \cos \pi/2 \cdot \sin u$

$1 \cdot \cos u - 0 \cdot \sin u = \cos u$ ✓

14) $\csc\left(\frac{\pi}{2} - u\right) = \sec u$

14) ____________

$$\frac{1}{\sin(\frac{\pi}{2} - u)} = \frac{1}{\sin \pi/2 \cos u - \sin u \cdot \cos \pi/2}$$

$$= \frac{1}{1 \cdot \cos u - \sin u \cdot 0} = \frac{1}{\cos u} = \sec u \checkmark$$